III. From the Cartesian equation of the Cassinian Oval, we deduce

$$y^2=\sqrt{(m^4+4c^2x^2)}-(c^2+x^2)\ldots\ldots(3),$$

an equation which gives all the *real* points of the Oval in consideration.

$$\therefore\left(\frac{dy}{dx}\right)^2=\frac{[4c^4-4c^2\sqrt{(m^4+4c^2x^2)}+m^4+4c^2x^2]x^2}{(m^4+4c^2x^2)[\sqrt{(m^4+4c^2x^2)}-(c^2+x^2)]}\ldots\ldots(\gamma).$$

Representing the semi-axis major of the Cassinian Oval by $a,=\sqrt{(m^2+c^2)}$, we have

$$\mathsf{P}=4\left(\frac{2c^2-m^2}{m^2\sqrt{(m^2-c^2)}}\right)\int_0^a\sqrt{\left(\frac{m^4(m^2-c^2)}{(2c^2-m^2)^2}+x^2\right)}dx$$

$$=2\left[\left(\frac{\sqrt{[m^4(m^2-c^2)+(m^2+c^2)(2c^2-m^2)^2]}}{m^2}\right)\sqrt{\left(\frac{m^2+c^2}{m^2-c^2}\right)}\right.$$

$$+\frac{m^2\sqrt{(m^2-c^2)}}{2c^2-m^2}$$

$$\left.\log\left(\frac{(2c^2-m^2)\sqrt{(m^2+c^2)}+\sqrt{[m^4(m^2-c^2)+(m^2+c^2)(2c^2-m^2)^2]}}{m^2\sqrt{(m^2-c^2)}}\right)\right],$$

$$=\tfrac{16}{9}\left[\frac{3\sqrt{(106)}}{5}+\tfrac{5}{3}\log\left(\frac{9+\sqrt{(106)}}{5}\right)\right]=14.9833,$$

when $m^2=5$ and $c^2=4$.

[To be continued.]

POSTULATE II. OF EUCLID'S ELEMENTS.

By Professor JOHN N. LYLE, Ph. D., Westminister College, Fulton, Missouri.

"Let it be granted that a terminated straight line may be produced to any length in a straight line."

Euclid lays down the statement just quoted as his second postulate regulative of geometrical constructions. Wherever in unbounded space any point may be located to which a straight line has been extended, Euclid assumes that the straight line may be lengthened out beyond that point.

Riemann assumes that every straight line is finite in length, and if extended will ultimately return to the starting point.

If a straight line that is produced from a given point eventually returns to the same point, Euclid's postulate 2 is false.

On the other hand, if the second postulate of Euclid is true, the Rie-

mannian hypothesis that contradicts it must be false. This follows inevitably by the logical law of *Excluded Middle*, according to which if one of two propositions that mutually contradict each other is true, the other must be false.

According to the Euclidian view the longer a straight line is the further apart are its ends.

According to the Riemannian view a straight line may be lengthened until its ends approach and ultimately meet.

The hypothesis of Riemann and the 2nd postulate of Euclid contradict each other. Hence, both cannot be true. To accept both is to discredit logical law. To say that we do not know which is true is to confess that we are not in possession of geometrical Science.

According to the laws of logical deduction, if Euclid's postulate 2 is false, the geometrical System derived from it is not true.

On the other hand, if the assumption that contradicts Euclid's postulate 2 is false, the system logically deduced from it is not true. Sound geometrical propositions are not obtained by logical deduction from false data.

According to the Riemannian hypothesis the angle sum of a rectilineal triangle is greater than two right angles. But Lobatschewsky proves in his theorem 19 that the angle sum of a rectilineal triangle cannot be greater than two right angles. The hypotheses of Lobatschewsky and Riemann, therefore, are seen to clash with each other as well as with the axioms, postulates and theorems of Euclid's Elements.

The chords of arcs of circles are not identical with the arcs subtended by them. Hence *rectilineal* triangles should not be treated as identical with *spherical* triangles. This statement holds whatever the length of the radius of the sphere may be. The radius of every sphere has *two* ends, one at the centre and the other at the surface. But every straight line with *two* ends is *finite.* We are now face to face with Postulate III. of Euclid's Elements.

SUBSTITUTION GROUPS.

THE CONSTRUCTION OF INTRANSITIVE GROUPS CONTINUED.

Before seeking all of the possible intransitive groups of degree* six it seems well to call attention to several facts which may be employed to advantage in this work. To illustrate we shall employ a group which was given before, viz.

* The degree of a group is equal to the number of letters it involves. Thus $(abcd)$ pos is of the fourth degree.